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# NUMERICAL SOLUTION OF THE BOUNDARY VALUE PROBLEM FOR THREE-PHASE FILTRATION IN POROUS MEDIA

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**Abstract.** This article presents a numerical solution of a boundary value problem for three-phase filtration in porous media, focusing on the oil–water–gas system in a gas-bearing reservoir. The mathematical model is formulated on the basis of filtration theory and includes continuity equations, phase motion equations, state relations, capillary pressure functions, and relative permeability coefficients. Since the obtained system of nonlinear parabolic differential equations is analytically difficult to solve, a finite-difference approach combined with iterative methods is applied. In the proposed scheme, reservoir pressure is calculated implicitly, while the saturation of oil, water, and gas is determined explicitly at each time layer. The resulting three-diagonal algebraic system is solved by the sweep method with an appropriate convergence criterion. A computational algorithm and MatLab-based software were developed to implement the proposed model. Numerical experiments were conducted for a gas field with specified reservoir parameters, well locations, and production rates. The results show changes in pressure distribution and phase saturation over time. The developed approach can be used for forecasting and analyzing oil and gas field development processes.

***Keywords:*** *multiphase filtration, porous media, finite-difference method, iterative method, oil-water-gas system, numerical simulation, relative permeability, capillary pressure.*

## Introduction.

1. Mathematical models of multiphase filtration of liquid and gas in porous media are based on the general laws of hydrodynamics and are represented as systems of nonlinear complex parabolic differential equations. Obtaining analytical solutions to such complex systems of differential equations is usually impossible. Therefore, it is desirable to develop generalized mathematical models, efficient numerical methods, algorithms, and software that can be effectively used for the analysis and forecasting of oil and gas fields.

Mathematical modeling of filtration processes occurring in the strata of oil and gas fields is currently especially important for the analysis and forecasting of processes. Mathematical modeling in the design of new oil and gas fields, forecasting existing ones by their main indicators, control and analysis of development serves to increase the productivity of fields.

Currently, mathematical models describing the joint filtration of liquids and gases in porous media are well known. Significant contributions to the development of these models were made by H. Aziz, E. Sattari, E.S. Zakirov, N.N. Veregin, V.N. Nikolaevsky, V.M. Shestakov, F.B. Abutaliev, D.F. Fayzullaev, R. Sadullaev, and other scientists

[1-3]. The results of in-depth study of these models and necessary research show that they cannot be considered fully studied at present. Numerical simulation methods effectively applied on modern computers are a new, efficient tool for basic scientific research. In this case, numerical simulation becomes not only a means of obtaining quantitative characteristics, but also a method for studying the process under investigation. Thus, based on the given physical model covering the main aspects of the process, it is possible to obtain a corresponding adequate mathematical model in the form of a system of numerically solvable equations using a personal computer.

It is known that in the process of mathematical modeling for solving any practical problem on a computer, the following sequence is carried out: study of the object, formation of data related to the model - mathematical model - construction of a discrete model using numerical methods - creation of an algorithm - development of a computer program - conducting computational experiments - analysis of the result or comparison with experimental and other data. The computational part of this object chain of mathematical technology is: mathematical model - numerical algorithm - program - computer calculation. The use of this technology allows analyzing, forecasting and controlling non-stationary processes of oil and gas production.

The investigation of temporal fluid and gas movements within porous media is conducted by flooding oil or gas-bearing formations during extraction. Since the joint flow of multiphase liquids is highly complex and cannot be directly observed deep underground, the primary physical processes are studied using mathematical tools. Computer simulation remains an efficient and relatively inexpensive method for analyzing this phenomenon, making mathematical modeling highly relevant for this issue.

The development of oil and gas fields is a continuous technological process and, as an object of design and control, belongs to the class of large-scale systems. It is characterized by technical, economic, technological, and geological commonality of specific features [4, 8].

The subsystem for the development of oil and gas fields includes the following part of the overall production process: the sequence of physical filtration processes of gas, condensate, and water in a porous permeable layer, and the movement of these components through wells.

A literature review indicates that an automated system for forecasting key indicators of oil and gas fields with a user-friendly interface, capable of conducting qualitative computational experiments, is not yet fully developed. To address this problem, the following tasks must be accomplished:

• Development of mathematical models for the filtration of two- and three-phase liquids in porous media;

• Development of algorithms for solving problems of two- and three-phase liquid filtration in porous media;

• Development and execution of computational experiments using software to calculate key indicators for the development of oil and gas fields.

Problem statement. The equations of filtration theory are employed to describe three-phase filtration processes. For mathematical modeling of unstable filtration in a one- or two-dimensional setting for two- and three-phase liquids within a porous medium, the following filtration theory equations are utilized [6-9].

Equations of continuity:

- $div\left(\frac{1}{B_o}\vec{\upsilon}_o\right) = -\frac{\partial}{\partial t}\left(\frac{1}{B_o}mS_o\right) + q_o$ for the oil phase;

- $div\left(\frac{1}{B_w}\vec{\upsilon}_w\right) = -\frac{\partial}{\partial t}\left(\frac{1}{B_w}mS_w\right) + q_w$ for the aqueous phase;

-

$$div\left[\frac{R_s}{B_o}\vec{\upsilon}_o + \frac{1}{B_g}\vec{\upsilon}_g\right] = -\frac{\partial}{\partial t}\left[m\left(\frac{R_s}{B_o}S_o + \frac{1}{B_g}S_g\right)\right] + q_g + R_s \cdot q_o$$

for the gas phase.

Motion equation for each phase $\vec{\upsilon}_t = -\frac{KK_{rt}}{\mu_t}(grad\ P_t + \gamma_t\ gradz)$ .

State equation for each phase $\rho_i = f(P_i)$ .

In addition to these equations of filtration theory, the following relations are applied:

So+Sw+Sg=1, So+Sw=1, So+Sg=1,

Po-Pw=Pcow =f1(Sw, Sg), Pg-Po=Pcog =f2(Sw, Sg).

Here:

• Pcow и Pcog - capillary pressures for "oil+water" and "oil+gas" systems respectively;

• Bo, Bw, Bg - volume coefficients;

• $\vec{\upsilon}_o, \vec{\upsilon}_w, \vec{\upsilon}_g$ - filtration rate;

• So, Sw, Sg - saturation of oil, water, and gas respectively;

• $q_o$, $q_w$, $q_g$ - respectively the well flow rate of each phase;

• Rs - gas miscibility with oil;

• k - absolute permeability of a layer;

• Kl - relative conductivity;

• $\mu_i$ - viscosity l - is the viscosity of the liquid pertaining to the phase

• $P_l, \rho_l, \gamma_l$ - is the pressure, density, and specific weight corresponding to the phase;

• z - is the distance from a point to any plane.

During filtration, water, oil, and gas retain their phases; gas dissolves in oil, while water remains immiscible. It is assumed that the fluids within the layer are at a constant temperature and in thermodynamic equilibrium.

Mathematical models describing two- or three-phase filtration flow in a porous medium layer are constructed based on the aforementioned fundamental equations of filtration theory.

Research into oil filtration in porous media has shown that intensive operation of well galleries in deep zones reduces the oil recovery factor of reservoir systems. Meanwhile, these processes, governed by such equations, must be implemented in computational methods used for mathematical modeling. Solving these problems requires developing adequate mathematical models for oil field development, creating efficient algorithms for calculating key parameters, and utilizing high-performance computing systems. Undoubtedly, such efficient computational algorithms play a crucial role in studying oil filtration in porous media based on its primary indicators.

Two-phase filtration processes occurring in the porous media of oil fields are highly complex and generally characterized by the following features: the process is described by partial differential equations and cannot be solved analytically; the coefficients of the differential equation depend on temporal and spatial variable coordinates; permeability and reservoir thickness are not piecewise homogeneous; the porosity coefficient in the reservoir varies across different zones, etc. Consequently, this complicates the solution of the specific boundary value problem.

Taking these features into account, the unsteady two-phase oil filtration process in a porous medium is based on the following assumptions:

• The oil layer under consideration possesses a constant width and length L (forming a rectangular area) along with a porosity m;

• The initial reservoir pressure is denoted as Pn;

• The production rate Q derived from oil wells is assumed to be uniform on average;

• Oil flow within the porous medium is linear and adheres to Darcy's linear law;

• The physical properties of the oil remain constant over time;

• Permeability coefficients and formation thickness vary across different zones, indicating heterogeneity

**Mathematical Model.**

Based on the aforementioned fundamental models and hypotheses of filtration theory, a mathematical model for the multiphase filtration process in the reservoir is presented, alongside the corresponding boundary and initial conditions for the specific field.

In the study of three-phase filtration, such as in an oil-water-gas system, the function of reservoir pressure and the saturation levels of these components play a crucial role in evaluating field development. Furthermore, it is significant that relative permeability coefficients have been determined experimentally to a sufficient extent.

Since the mathematical model is nonlinear with respect to these parameters and relative conductivity, its numerical solution presents considerable complexity.

It is assumed that the porous space of the medium is predominantly filled with gas, and the liquid phase is initially in equilibrium at the start of the absorption process. The modeling employs continuity, state, and phase flow motion equations.

Assuming uniform reservoir properties throughout the formation, the three-phase filtration problem in the oil-water-gas system is formulated as a system of nonlinear parabolic differential equations with a one-dimensional mathematical structure. Consequently, the mathematical model of the problem is expressed by the following system of parabolic differential equations.

.

$$\begin{cases} \dfrac{\partial}{\partial x}\left[\lambda_{1w}\dfrac{\partial P_w}{\partial x}-\gamma_w\dfrac{\partial z}{\partial x}\right]=\dfrac{\partial}{\partial t}(m\rho_w S_w)+q_w, \\ \dfrac{\partial}{\partial x}\left[\lambda_{1o}\dfrac{\partial P_o}{\partial x}-\gamma_o\dfrac{\partial z}{\partial x}\right]=\dfrac{\partial}{\partial t}[m\rho_o(1-S_w-S_g)]+q_o; \\ \dfrac{\partial}{\partial x}\left[R_S\lambda_o\dfrac{\partial P_o}{\partial x}-\gamma_o\dfrac{\partial z}{\partial x}\right]+\dfrac{\partial}{\partial x}\left[\lambda_g\dfrac{\partial P_g}{\partial x}-\gamma_w\dfrac{\partial z}{\partial x}\right]= \\ =\dfrac{\partial}{\partial t}\left[mR_S\rho_o(1-S_g)+mS_g\rho_g\right]-R_S q_o\delta(x-\xi_i)+q_g; \\ S_o+S_w+S_g=1; \\ P_o-P_w=P_{cow};\quad P_g-P_o=P_{cog}. \end{cases}$$

(1)

Initial and boundary conditions:

$$P_o(x,0)=P_o^H(x),\quad P_g(x,0)=P_g^H(x),\quad P_w(x,0)=P_w^H(x);$$

(2)

$$S_o(x,0)=S_o^H(x),\quad S_g(x,0)=S_g^H(x),\quad S_w(x,0)=S_w^H(x);$$

(3)

$$-\frac{k}{\mu_o}\frac{\partial P_o}{\partial x}=\alpha\left(P_{Ao}-P_o\right),\quad x=0;\qquad \frac{k}{\mu_o}\frac{\partial P_o}{\partial x}=\alpha\left(P_{Ao}-P_o\right),\quad x=L$$

(4)

$$-\frac{k}{\mu_g}\frac{\partial P_g}{\partial x}=\alpha\left(P_{Ag}-P_g\right),\quad x=0;\qquad \frac{k}{\mu_g}\frac{\partial P_g}{\partial x}=\alpha\left(P_{Ag}-P_g\right),\quad x=L$$

(5)

$$-\frac{k}{\mu_w}\frac{\partial P_w}{\partial x}=\alpha\left(P_{Aw}-P_w\right),\quad x=0;\qquad \frac{k}{\mu_w}\frac{\partial P_w}{\partial x}=\alpha\left(P_{Aw}-P_w\right),\quad x=L$$

(6)

Assume that oil and water are incompressible, meaning $\rho_o=const$ and $\rho_w=const$, while gas is compressible, with its density given by the equation of state $\rho_g=P_g/RTZ$. Applying these relations along with $\rho_g-\rho_o=\rho_{cog}$, we can express the third equation of the system as follows.

$$\frac{\partial}{\partial x}\left[R_s\lambda_o\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}-\gamma_o\frac{\partial z}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left[\lambda_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{\partial z}{\partial x}\right)\right]=$$
$$=\frac{\partial}{\partial t}\left[mR_s\rho_o\left(1-S_g-S_w\right)+m\frac{P}{RTZ}S_g\right]+R_s q_o+q_g$$

In the boundary value problem, we move to dimensionless variables using the following formulas.

$$x^*=\frac{x}{L};\quad k^*=\frac{k}{k_x};\quad h^*=\frac{h}{h_x};\quad P_g^*=\frac{P_g}{P^H};\quad P_o^*=\frac{P_o}{P^H};\quad P_w^*=\frac{P_w}{P^H};$$

$$P_{cow}^*=\frac{P_{cow}}{P^H};\quad P_{cog}^*=\frac{P_{cog}}{P^H};\quad \tau=\frac{P^H k_x t}{m\mu_o L^2};\quad q_i^*=\frac{\mu_o L^2}{k_x h_x P^H 2\pi}\mathrm{q_i};\quad \alpha^*=\alpha\frac{\mu_o L}{k_x}.$$

The characteristic values of permeability and capacitance for layer $k_x, h_x$ are shown here; $p^H$ represents the characteristic pressure value; $L$ denotes the layer length; and α is a dimensional parameter.

As a result, the boundary value problem (1)-(6) is reduced to a dimensionless problem according to these formulas. Then, after substitutions, we remove the asterisks appearing in the system of equations and obtain the following system of equations.

$$\frac{\partial}{\partial x}\left[K_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]=\frac{\partial S_o}{\partial\tau}\frac{\mu_o L^2}{K\rho_o P_H}q_o\ ;$$

$$\frac{\partial}{\partial x}\left[K_w\left(\frac{\partial P_w}{\partial x}-\gamma_w\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]=\frac{M_w}{M_o}\frac{\partial S_o}{\partial\tau}+\frac{\mu_w L^2}{K\rho_w P_H}q_w\ ;$$

$$\frac{\partial}{\partial x}\left[R_s K_o\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}\frac{\gamma_o L}{P_H}\frac{\partial z}{\partial x}\right)\right]+\frac{P_H\mu_o}{\rho_o RTZ\mu_g}\frac{\partial}{\partial x}\left[K_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]=$$
$$=\frac{P_H}{RTZ\rho_o}\frac{\partial(PS_g)}{\partial\tau}+\frac{\partial}{\partial\tau}\left[R_s\left(1-S_w-S_g\right)\right]+\frac{\mu_o L^2}{K\rho_o P_H}(R_s q_o+q_g)\ .$$

Dividing the second equation of the system by expression $\frac{\mu_w L^2}{K\rho_w P_H}$ and multiplying by expression $\frac{\mu_o L^2}{K\rho_o P_H}$, then adding the equations, yields the following generalized equation:

$$\frac{\partial}{\partial x}\left[K_o\left(\frac{\partial P_o}{\partial x}-\gamma_o\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{\mu_o}{\mu_w}\frac{\rho_w}{\rho_o}\frac{\partial}{\partial x}\left[K_w\left(\frac{\partial P_w}{\partial x}-\gamma_w\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{\partial}{\partial x}\left[R_s K_o\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}\frac{\gamma_o L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{P_H\mu_o}{\rho_o RTZ\mu_g}\frac{\partial}{\partial x}\left[K_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]=$$
$$=\frac{P_H}{RTZ\rho_o}\frac{\partial(PS_g)}{\partial\tau}+\frac{\partial S_o}{\partial\tau}+$$
$$+\frac{\mu_w}{\mu_o}\frac{\partial S_w}{\partial\tau}+\frac{\partial}{\partial\tau}\left[R_s\left(1-S_w-S_g\right)\right]+$$
$$+\frac{\mu_o L^2}{K\rho_o P_H}(q_o+R_s q_o+q_g+q_w)\ .$$

Using the relationships $P_o=P_g-P_{cog}$ and $P_w=P_g-(P_{cog}+P_{cow})$ given above, we write the equation as follows.

$$\frac{\partial}{\partial x}\left[K_o\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}-\frac{\gamma_o L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{\mu_o}{\mu_w}\frac{\rho_w}{\rho_o}\frac{\partial}{\partial x}\left[K_w\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}-\frac{\partial P_{cow}}{\partial x}-\gamma_w\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{\partial}{\partial x}\left[R_s K_o\left(\frac{\partial P_g}{\partial x}-\frac{\partial P_{cog}}{\partial x}-\gamma_o\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]+$$
$$+\frac{P_H\mu_o}{\rho_o RTZ\mu_g}\frac{\partial}{\partial x}\left[K_g\left(\frac{\partial P_g}{\partial x}-\gamma_g\frac{L}{P_H}\frac{\partial z}{\partial x}\right)\right]=$$
$$=\frac{P_H}{RTZ\rho_o}\frac{\partial(PS_g)}{\partial\tau}+\frac{\partial S_o}{\partial\tau}+\frac{\mu_w}{\mu_o}\frac{\partial S_w}{\partial\tau}+$$
$$+\frac{\partial}{\partial\tau}\left[R_s\left(1-S_w-S_g\right)\right]+$$
$$+\frac{\mu_o L^2}{K\rho_o P_H}(q_o+q_w+R_s q_o+q_g)\ .$$

Numerical solution approach. Computational techniques are increasingly employed to analyze mathematical models of hydrodynamic systems. Nevertheless, applying these methods to solve certain nonlinear equations in filtration theory presents significant challenges. These obstacles stem primarily from the equations' nonlinearity and the high complexity of the computational algorithms involved.

To address the equation, we adopt an implicit scheme for determining pressure functions and an explicit scheme for calculating saturation values. Consequently, straightforward elimination and iteration techniques are utilized during computation.

We construct a discrete model and computational scheme based on the finite difference method. In this context, the resulting system of three-point finite difference algebraic equations is nonlinear with respect to the pressure function, as well as saturation and relative permeability coefficients. Thus, its resolution relies on a combination of finite difference and iterative methods.

The final equation is solved for gas pressure. To numerically simulate the boundary value problem, we establish the following grid:

$$D_{h,\tau_k}=\left\{\begin{array}{l}(x_i,t_k): x_i=x_{i-1}+\Delta x,\\ i=\overline{1,n},\ \ x_0=0,\ \ x_N=1,\\ t_k=t_{k-1}+\Delta\tau,\ \ k=1,2,...\\ t_0=0\end{array}\right\}.$$

We select the system step size so that nodes align with well locations.

By solving the specified boundary value problem numerically, we approximate the last generalized equation along with its associated boundary condition, yielding the subsequent system of three-point finite difference equations.

$$\left\{\begin{array}{l}\left(3-2\Delta xL\alpha\right)P_{g0}+4P_{g1}-P_{g2}=2\Delta x\alpha LP_A,\\ a_i P_{g\,i-1}-b_i P_{g\,i}+c_i P_{g\,i+1}=-d_i\quad i=1,2,...,N-1,\\ \left(3-2\Delta xL\alpha\right)P_{g\,N}+4P_{gN-1}-P_{g\,N-2}=-2\Delta x\alpha LP_A;\end{array}\right. \qquad (7)$$

Here

$$P_{gi}=P_g(x_i,t_{k+1});$$
$$A_i=\frac{\mu_o}{\mu_w}\frac{\rho_w}{\rho_o}\tilde{K}_{wi-0.5}+\tilde{K}_{oi-0.5}+$$
$$+\left(\tilde{R}_{si-0.5\,j}\tilde{K}_{oi-0.5}\right)+\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left(\tilde{K}_{gi-0.5}\tilde{P}_{gi-0.5}\right);$$

$$C_i = \frac{\mu_o}{\mu_w}\frac{\rho_w}{\rho_o}\tilde{K}_{wi+0.5} + \tilde{K}_{oi+0.5} + $$
$$+\left(\tilde{R}_{si+0.5}\tilde{K}_{oi+0.5}\right) + \frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left(\tilde{K}_{gi+0.5}\tilde{P}_{gi+0.5}\right);$$
$$B_i = A_i + C_i + \frac{h_1^2}{\tau}\frac{P_H}{\rho_o RTZ}\tilde{S}_{gi};$$

$$F_i = \frac{h_1^2}{\tau}\left\{\frac{P_H}{\rho_o RTZ}\left(\bar{P}_{gi}\bar{S}_{gi}\right) - \left[\begin{array}{l}\tilde{R}_s\left(1-\tilde{S}_{gi}-\tilde{S}_{wi}\right)- \\ -\bar{R}_s\left(1-\bar{S}_{gi}-\bar{S}_{wi}\right)\end{array}\right] - \right.$$
$$\left. -\left[\left(1-\tilde{S}_{wi}-\tilde{S}_{gi}\right)-\left(1-\bar{S}_{wi}-\bar{S}_{gi}\right)\right] - \frac{\rho_w}{\rho_o}\left(\tilde{S}_{wi}-\bar{S}_{wi}\right)\right\} -$$
$$-\frac{\mu_o}{\mu_w}\frac{\rho_w}{\rho_o}\left\{\tilde{K}_{wi+0.5}\left[\begin{array}{l}\tilde{P}_{cogi+1}-\tilde{P}_{cogi}+\tilde{P}_{cowi+1}- \\ -\tilde{P}_{cowi}+\gamma_w\frac{L}{P_H}\left(Z_{i+1}-Z_i\right)\end{array}\right] - \right.$$
$$-\tilde{K}_{wi-0.5}\left[\begin{array}{l}\tilde{P}_{cogi}-\tilde{P}_{cogi-1}+\tilde{P}_{cowi}- \\ -\tilde{P}_{cowi-1}+\gamma_w\frac{L}{P_H}\left(Z_i-Z_{i-1}\right)\end{array}\right] -$$
$$-\tilde{K}_{oi+0.5}\left[\tilde{P}_{cogi+1}-\tilde{P}_{cogi}+\gamma_o\frac{L}{P_H}\left(Z_{i+1}-Z_i\right)\right] +$$
$$+\tilde{K}_{oi-0.5}\left[\tilde{P}_{cogi}-\tilde{P}_{cogi-1}+\gamma_o\frac{L}{P_H}\left(Z_i-Z_{i-1}\right)\right] -$$
$$-\left(\tilde{R}_{si+0.5}\tilde{K}_{oi+0.5}\right)\left[\tilde{P}_{cogi+1}-\tilde{P}_{cogi}+\gamma_o\frac{L}{P_H}\left(Z_{i+1}-Z_i\right)\right] +$$
$$\left(\tilde{R}_{si-0.5}\tilde{K}_{oi-0.5}\right)\left[\tilde{P}_{cogi}-\tilde{P}_{cogi-1}+\gamma_o\frac{L}{P_H}\left(Z_i-Z_{i-1}\right)\right] -$$
$$-\frac{\mu_o}{\mu_g}\frac{P_H}{\rho_o RTZ}\left\{\begin{array}{l}\left(\tilde{K}_{g+0.5}\tilde{P}_{g+0.5}\right)\left[\gamma_g\frac{L}{P_H}\left(Z_{i+1}-Z_i\right)\right]- \\ -\left(\tilde{K}_{gi-0.5}\tilde{P}_{gi-0.5}\right)\left[\gamma_g\frac{L}{P_H}\left(Z_{i+1}-Z_{i-1}\right)\right]\end{array}\right\}.$$

Here, $\tilde{P},\tilde{S},\tilde{K}$ denotes the approximate values of reservoir pressure, saturation, and relative permeability determined during the iteration process. For the value $\tilde{P}_i^{(s)}$ in the k-th time layer, the zeroth approximation is taken. The values of reservoir pressure, saturation, and $\tilde{P}_i^{(0)} = P_i$, $\tilde{K} = K(\tilde{S})$; $\bar{P},\bar{S},\bar{K}$ relative permeability in the k-th time layer.

The system of finite-difference algebraic equations (7) has three diagonals. Therefore, its solution can be obtained by the left sweep method in each iteration layer s. The condition for terminating the iteration process at each *i* is as follows:

$$\max_{0\le i\le N}\left|P_{gi}^{(s)} - P_{gi}^{(s-1)}\right| \le \varepsilon_p,$$

where $\varepsilon_p$ is the accuracy of the iterative process.

To determine the saturation of oil and water, as well as the pressure in the oil and water phases, we perform calculations using the following formulas.

$$P_{oi} = P_{gi} - P_{cogi}, \quad P_{wi} = P_{gi} - (P_{cogi} + P_{cowi})$$

Based on these formulas, according to Darcy's law, we determine the fraction of oil and water in the total flow using the following formulas:

$$q_o = \frac{KK_o}{\mu_o}\left\{\frac{dP_o}{dx} - \gamma_o\frac{dz}{dx}\right\};$$
$$q_w = \frac{KK_w}{\mu_w}\left\{\frac{dP_w}{dx} - \gamma_w\frac{dz}{dx}\right\}.$$

Approximating the first and second equations of system (1), we use the following expressions to determine the saturation of water and oil.

$$S_{o_i}^{(\mathrm{m})} = S_{o_i}^{(\mathrm{m}-1)} + \frac{\tau}{h\cdot h_1}\left\{K_{oi+0.5}\left[\begin{array}{l}P_{oi+1}-P_{oi}- \\ -\gamma_o\frac{L}{P_H}\left(Z_{i+1}-Z_i\right)\end{array}\right] - \right.$$
$$\left. -K_{oi-0.5}\left[\begin{array}{l}P_{oi}-P_{oi-1}- \\ -\gamma_o\frac{L}{P_H}\left(Z_i-Z_{i-1}\right)\end{array}\right]\right\} - \tau\frac{\mu_o L^2}{K\rho_o P_H}q_o.$$

Values *K* and *P* are taken from the determined temporal layer *k+1*, while the value - $\bar{S}_{oij}$ is taken from the k-th temporal layer.

The termination criterion for the iterative process is evaluated by the relation

$$\left\{\max_{0\le i\le N}\left|S_{o_i}^{(m)} - S_{o_i}^{(m-1)}\right|, \quad \max_{0\le i\le N}\left|S_{w_i}^{(m)} - S_{w_i}^{(m-1)}\right|\right\} \le \varepsilon_s$$

where - $\varepsilon_s$ is the degree of iteration accuracy, *m* - is the iteration number.

Gas saturation is determined according to the following formula:

$$S_{g_i} = 1 - S_{w_i} - S_{o_i}, \quad i = 0,1,2,...,\mathrm{N}.$$

In subsequent time layers, these calculations are performed in the same sequence.

**Brief algorithm of the calculation process.** Determination of the main indicators of gas field development. The numerical implementation of the

solution algorithm for a given time interval is carried out in the following stages:

1. Reading input data from a file (input):

grid size n; number of wells *nq*; time step *dt* (day); end of calculation *tcon* (day); layer length *L*; layer thickness *h* and environmental parameters: porosity coefficient *m*, permeability coefficient *k*, initial pressure $p_n$, saturation *S*, density $\rho$, etc.

2. Calculation of the coefficients of the finite-difference equation (7).

$$a_i, b_i, c_i, d_i \ (i = 2,..,N-1);$$

3. Determination of initial values of control coefficients $A_0$ and $B_0$ from boundary conditions;

4. Calculation of values of control coefficients $A_i$ and $B_i$ *(i = 1, 2,…, N-1)*;

5. The final value of the pressure function $P_n$ is determined from the boundary conditions;

6. Values of the pressure function are calculated using backward pass.

7. The iteration process is checked. If the iteration process is successful, a transition to the next time layer will be made. Otherwise, the process will repeat.

This algorithm is repeated in each time layer. In this case, each obtained solution serves as the initial condition for the next time step.

**Computational experiments.**

Software for the computational algorithm designed to solve the boundary value problem (1)-(4) has been developed using MatLab. High reliability of initial data is required for conducting computational experiments. In this direction, when solving boundary value problems of multiphase liquid filtration, capillary pressure and relative phase permeability play an important role. In practice, capillary pressure is considered as some dimensionless function of saturation.

$K_w = 8.59667\, S_w^3 + 9.16878\, S_w^2 + 3.3075\, S - 0.362522;$

$K_o = -6.16392\, S_w^3 + 9.61207\, S_w^2 - 5.16941 S_w + 1.00871.$

When solving the problem, the initial data were obtained as follows: reservoir length *L=10000 m*, initial reservoir pressure *P=300 atm*., oil viscosity $\mu = 3sP$; gas viscosity $\mu = 0.02sP$; It was assumed that for 1080 days from the beginning of the development of the gas field. Computational experiments are presented in graphs in the figure below.

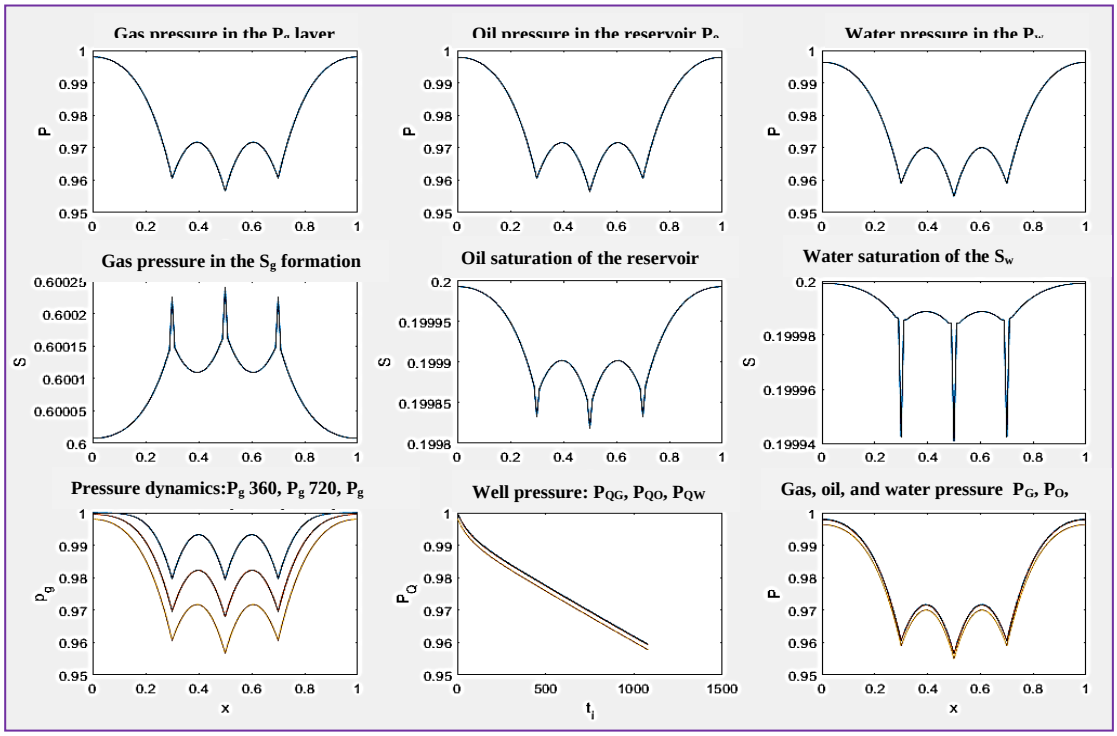


*Fig. 1. Plots depicting pressure and saturation variations within the reservoir.*

Three wells are situated near the center along the x-coordinate line in the gas well. They are located at coordinates $x = 0,5$ and $x = 0,3,\ 0,71$, respectively, and share an identical production rate: $Q_1 = 600,000\ m^3/day$; $Q_{2,3} = 600,000\ m^3/day$. Other initial parameters were defined as follows: the absolute permeability coefficient of the formation is $k = 0,2$; and the porosity coefficient is $m = 0,1$. The figure illustrates the pressure and saturation distribution profiles within the reservoirs. One can observe the pressure distribution in the surrounding medium resulting from pressure depletion in the wells. Furthermore, as gas saturation increases, there is a corresponding decrease in oil saturation and water saturation in the outer phase.

**Conclusion.**

Computational experiments examined multiphase filtration processes to determine key parameters of gas fields, including pressure and saturation levels. The developed methods and software for calculating primary gas field parameters can be applied during the design phase, as well for analyzing and forecasting development processes in oil and gas fields.

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